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Finding Core Motions from Magnetic Observations [and Discussion]

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Finding core motions from magnetic observations

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It is possible to infer general properties of fluid flow in the core, but there is a large class of flows that fit even perfect data. This class can be reduced in size by making simplifying hypotheses about the flow and electrical conductivity of fluid in the core. For a hypothesis to be reasonable it must be possible to test it against observation; if the test proves favourable the hypothesis is adopted. Three such hypotheses are suggested here: the familiar one of perfect electrical conductivity, and two new ones, those of purely toroidal motion, and steady motion. The first two make it possible to find one component of core flow everywhere. In principle the third hypothesis allows both components to be found. A map of one component for the period 1959–74 shows significant north–south flow over Indonesia and rapid motion in a swath running from beneath North America, across Africa and down to the southern Indian Ocean. With longer-term data it may be possible to find both components of the flow.

1. Introduction

The slow changes in the magnetic field that take place on time scales longer than about 5 years are called the secular variation and are attributed to the inductive effects of fluid motion in the core. The secular variation therefore carries information about core motions, and this paper deals with how to extract this information.

In the core the magnetic field satisfies the induction equation

$$\dot{\boldsymbol{B}} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}, \tag{1}$$

where $\boldsymbol{v}$ is the flow, $\boldsymbol{B}$ the magnetic field and η the magnetic diffusivity. Roberts & Scott (1965) argue that η may be taken as zero for timescales of a few years or less because the diffusion time $\tau_\eta = L^2/\eta$ is about 12000 years for $L = 10^6$ m and is much longer than the advection time, $\tau_v = L/v$, which is only 30 years. Backus (1968) gives a more quantitative discussion and stresses that the hypothesis will be valid for sufficiently large anomalies, but must ultimately fail, and the question is therefore whether it is possible to resolve such small wavelengths with the available data. Diffusion cannot be ignored in a boundary layer near the core boundary and therefore the core flow should be interpreted as taking place at the top of the free stream, below the boundary layer.

Horizontal components of $\boldsymbol{B}$ may be discontinuous across the boundary layer but the radial component is continuous. Attention is therefore restricted to the radial component of (1) which gives, at the core surface where $\boldsymbol{v} \cdot \hat{\boldsymbol{r}} = 0$,

$$\dot{B} + \nabla \cdot (\boldsymbol{v}B) = 0, \tag{2}$$

where $B = \boldsymbol{B} \cdot \hat{\boldsymbol{r}}$.

The 'frozen-flux' hypothesis was tested against magnetic observations by Booker (1969). Equation (1) implies that integrals of $\dot{B}$ over patches of the core surface bounded by null flux curves (contours of zero B) are zero. If the integrals are found to be zero within the rather large errors involved, then one can proceed to calculate bounded flows satisfying (2), but the problem is non-unique and many such flows are admissible. In order to reduce the ambiguity two further hypotheses are introduced, those of purely toroidal and steady flow. Each imposes further constraints upon the data, providing tests of the corresponding hypothesis.

Statistical tests require proper estimates of the errors in the magnetic field and secular variation models. The models are in the form of continuous functions or spherical harmonic coefficients, which have been continued downward to the core–mantle boundary, and no error estimates have yet been made. To give some idea of the range of possible values, different field models for 1965 are compared. These are the IGRF, DEFLO2, the definitive model (Barraclough *et al.* 1978) as modified by Whaler (this symposium), and LSMF65 derived from annual means of three components from 106 stations for the period 1959–74, using the method of Shure *et al.* (1982). A full description of these models is in preparation.

2. Zero resistivity

Consider the frozen-flux hypothesis first. Integrating (2) over a patch of the core surface bounded by a contour of B gives

$$\int_{S_i} \dot{B}\,\mathrm{d}S = -B_i \int_{S_i} \nabla_{\mathrm{h}} \cdot \boldsymbol{v}\,\mathrm{d}S, \tag{3}$$

where ∇_{h} denotes horizontal divergence, and B_i the value of B on the boundary. For null flux curves $B_i = 0$ and

$$F_i = \int_{S_i} \dot{B}\,\mathrm{d}S = 0. \tag{4}$$

These necessary and sufficient conditions were derived by Backus (1968). For most current models of the main field there are five null-flux curves giving six distinct areas S_i, only five of which are independent. Hide (1981) shows that the unsigned flux integral

$$N = \oint |B|\,\mathrm{d}S$$

is independent of time, since

$$\mathrm{d}N/\mathrm{d}t = \sum_i \int_{S_i} \dot{B}\,\mathrm{sgn}\,(B)\,\mathrm{d}S, \tag{5}$$

and this has also been used as a test of the hypothesis. Individual integrals (4) give a better test.

Numerical values of F_i for two models are shown in table 1. Values for the small regions in the North Pacific and near the North Pole have been omitted. The values above and below the Equator confirm Booker's (1969) conclusion that the decay of the dipole provides a dominant contribution. The values are smaller for the IGRF, which is based on a more extensive data set, and it is generally true that these integrals are smaller for more accurate models. There is therefore no evidence of resistive effects in these models and the values of the integrals probably represent the noise level.

There are two ways to improve these tests of zero resistivity. One is by resolving shorter wavelengths by using, for example, Magsat data. The other is to use long-term data. Intensities have only been measured since 1825 but there is considerable information on direction from before that time. Directional data can be analysed to give a field model that has indeterminate total amplitude but is otherwise well defined (see, for example, Barraclough 1978). The amplitude factor can be found by assuming perfect conductivity and fitting the unsigned flux integral to the present value. It is then possible to calculate *intensities* from directional data. The intensities could be compared with archaeomagnetic measurements. Extrapolating the present rate of change of the dipole back to A.D. 1600 suggests a change in intensity of 20–30 % in some places, which is detectable by using archaeomagnetism and in excess of errors in the calculation of the unsigned flux integral.

3. Toroidal core flow

A purely toroidal flow has $\nabla_h \cdot \boldsymbol{v} = 0$. Equation (3) allows an estimate to be made of $\int_{S_i} \nabla_h \cdot \boldsymbol{v} \mathrm{d}S$, or the average upwelling over a surface. Near extrema of B the surface S_i approaches the limiting case of a point, and the upwelling can be found uniquely. Elsewhere the integral can be written as a line integral around a contour of B by using the two-dimensional divergence theorem

$$\int_{S_i} \nabla_h \cdot \boldsymbol{v} \mathrm{d}S = \oint v_n \mathrm{d}l, \tag{6}$$

where v_n is the component of $\boldsymbol{v}$ normal to the contour. This shows that the problem of finding v_n from such integrals is similar to that of finding seismic velocities from travel times, which are integrals of inverse velocities over ray paths. The difficulty here is that the integrals are over contours of B that do not intersect, and as every seismologist knows, intersecting paths are essential in resolving structure.

Whaler (1980) has shown that integrals of $\nabla_h \cdot \boldsymbol{v}$ are smaller than one would expect from a random field model, suggesting that the poloidal flow is small. Without making error estimates it is impossible to assert that the model can be explained by purely toroidal flow. However, the integrals can be compared with those over null-flux curves. If we adopt the frozen-flux hypothesis, which is widely accepted, then these integrals represent uncertainty in the model. Numerical values for some anomalies in the main field (figure 1) are displayed in table 1. There is no evidence from these numbers that these integrals are any larger than those for null-flux curves, even when the area of each integral is taken into account. Under Africa there is an anomaly with a large integral for a small area. This is a region where upwelling would appear strongest, but both field and secular variation contours are very complex there.

These results suggest that poloidal flow can be neglected as a first approximation. Writing (2) as

$$\dot{B} + \boldsymbol{v} \cdot \nabla_h B = -B \nabla_h \cdot \boldsymbol{v} \tag{7}$$

and treating the upwelling term on the right-hand side as an error (7) gives an estimate for v_n everywhere:

$$v_n = -\dot{B}/|\nabla_h B|. \tag{8}$$

The flow along the contour is indeterminate so that the new hypothesis has not resolved the ambiguity. The importance of this ambiguity can be illustrated in the simple case of westward drift. Suppose that the field pattern is observed to rotate steadily with angular velocity ω so that

$$\dot{B} = -\omega \, \partial B/\partial \phi,$$

then (7), with $\nabla_h \cdot \boldsymbol{v} = 0$, gives

$$v_\phi \frac{\partial B}{\partial \phi} + v_\theta \sin\theta \frac{\partial B}{\partial \theta} = \omega c \sin\theta \frac{\partial B}{\partial \phi}.$$

The solution has particular integral

$$v_\phi = \zeta = \omega c \sin\theta,$$

which is simple rotation of the core, and complementary function, (η_0, ζ_0), satisfying $\nabla_h \cdot \boldsymbol{v} = 0$, given by

$$(\eta_0, \zeta_0) = k\nabla_h \times (\hat{\boldsymbol{r}} B). \tag{9}$$

Equation (9) represents the ambiguity or *annihilator*.

Table 1. Integrals of B

region	contour value/MT	LSMF 65 integral	LSMF 65 area/c^2	IGRF 1965.0 integral	IGRF 1965.0 area/c^2
north of Equator	0	+761	5.84	635	5.74
south of Equator	0	−406	5.97	−497	6.06
off South Africa	0	−475	0.34	−289	0.34
off South America	0	+ 60	0.25	169	0.28
U.S.A.	0.3	+328	0.79		
	0.4	− 50	0.42		
	0.5	−111	0.18		
Mongolia	0.5	+188	0.25		
	0.6	+123	0.13		
Africa	−0.4	+192	0.12		

One problem frequently posed in connection with westward drift is whether or not it implies mass motion of core fluid. This result shows clearly that it is impossible to determine mass rotation from kinematic considerations of the field. The dominant contribution to B is the dipole $-2(a/c)^3 g_1^0 \cos\theta$, which when substituted into (9) gives

$$\zeta_0 = 2k(a/c)^3 g_1^0 \sin\theta.$$

This is again rotation of the core. k can take any value and therefore any rotation of the core is permissible, including no rotation at all.

Returning to (8), it will not be possible to derive bounded flows because errors ensure that $\dot{B}$ is not exactly zero when $|\nabla_h B|$ vanishes. Equation (8) presents problems wherever $|\nabla_h B|$ is small because errors are amplified there. Damped least squares provide a more stable procedure. v_n is chosen so as to minimize

$$E = \oint \{\dot{B} + v_n |\nabla_h B|\}^2 \, dS \tag{10}$$

subject to the constraint

$$\oint v_n^2 \, dS \leqslant 4\pi \bar{v}^2. \tag{11}$$

In practice v_n is expanded in spherical harmonics to very high order (usually 14 or 20) and $\bar{v}$ is chosen so as to provide a satisfactory final value of E. Carrying out the minimization with Lagrange multipliers in the usual way gives the normal equations

$$(S + \Lambda)\, \boldsymbol{u} = \boldsymbol{b}, \tag{12}$$

where $\boldsymbol{u}$ is a vector of unknown velocity coefficients, $-\boldsymbol{b}$ has elements

$$\oint \dot{B} |\nabla_h B| P_p^q(\cos\theta) \begin{Bmatrix}\sin\\ \cos\end{Bmatrix} q\phi \, dS,$$

S is a matrix with elements

$$\oint |\nabla_h B|^2 P_l^m(\cos\theta)\; P_p^q(\cos\theta) \begin{Bmatrix}\sin\\ \cos\end{Bmatrix} m\phi \begin{Bmatrix}\sin\\ \cos\end{Bmatrix} q\phi \,\mathrm{d}\Omega,$$

and Λ is a diagonal matrix with elements $4\pi\lambda/(2l+1)$ for Schmidt quasi-normalized spherical harmonics.

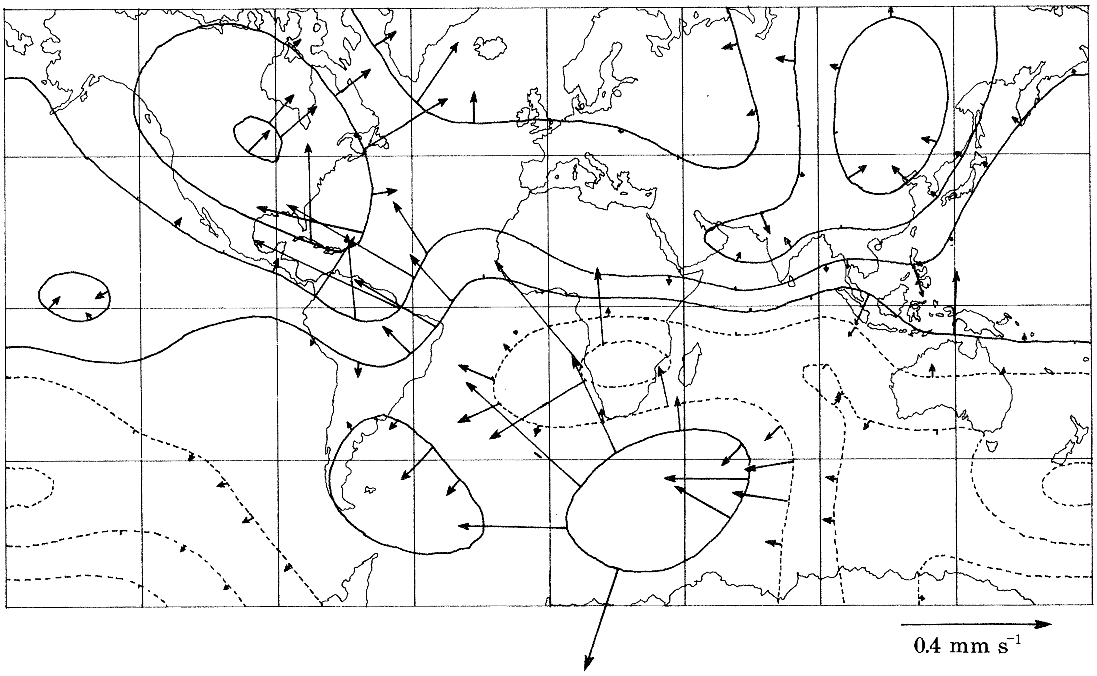

FIGURE 1. Contours of the vertical component of magnetic field, for model LSMF65, at an interval of 0.2 mT. Dashed contours are negative, the zero contour is solid. The small contour in the Pacific is a null-flux curve. Arrows give the component of flow perpendicular to the contour for $\lambda = 0.01$; scale as shown.

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This procedure avoids the unpleasant properties of truncated spherical harmonic expansions and is very simple to implement. Other norms might be used in place of (11): minimizing the maximum error for example. The resulting flow will *not* be toroidal, because of errors. It is possible to compute a self-consistent toroidal flow by using the Roberts & Scott (1965) formalism, but it is more instructive to be able to see where upwelling occurs and therefore where our hypothesis breaks down.

The misfit, E, is expressed as a percentage reduction of $\oint \dot{B}^2 \mathrm{d}S$, and $\bar{v}$ as the mean flow in millimetres per second. By sampling one component of a unit vector with random orientation one would expect to measure a component with r.m.s. length $\pi^{-\frac{1}{2}}$, so $\pi^{\frac{1}{2}}\bar{v}$ should be interpreted as the r.m.s. flow velocity. Plotting E against $\bar{v}$ for various values of λ gives a trade-off curve.

Results are given in table 2. The fit is not perfect for $\lambda = 0$ because the velocity series had to be truncated. As expected the smooth model LSMF65 gives a better fit for the same flow speed. It is important that the velocity do not fit the data *too* well because the toroidal motion hypothesis implies that errors are substantial and the signal:noise ratio may be as low as 50%. The preferred value of λ is 0.01. For LSMF65, further smoothing reduced $\bar{v}$, but did not change the appearance of v_n at all. There are noticeable differences between $\lambda = 0.01$ and $\lambda = 0.02$ for DEFLO2.

A map for $\lambda = 0.01$ is shown in figure 1. The most noticeable feature is northward and westward flow over the Caribbean, central Atlantic and off southern Africa. These are also areas showing rapid secular change at the surface. Arrows associated with null-flux curves give the velocity independent of the toroidal flow hypothesis. The velocity corresponding to westward rotation of 0.2° per year is 0.4 mm/s.

Table 2. Trade-off parameters for two field models

	λ	$\bar{v}$	E (%)
LSMF65	0	0.29	93
	0.01	0.14	67
	0.02	0.10	57
DEFLO2	0	0.36	85
	0.01	0.22	67
	0.02	0.17	57
	0.06	0.10	36

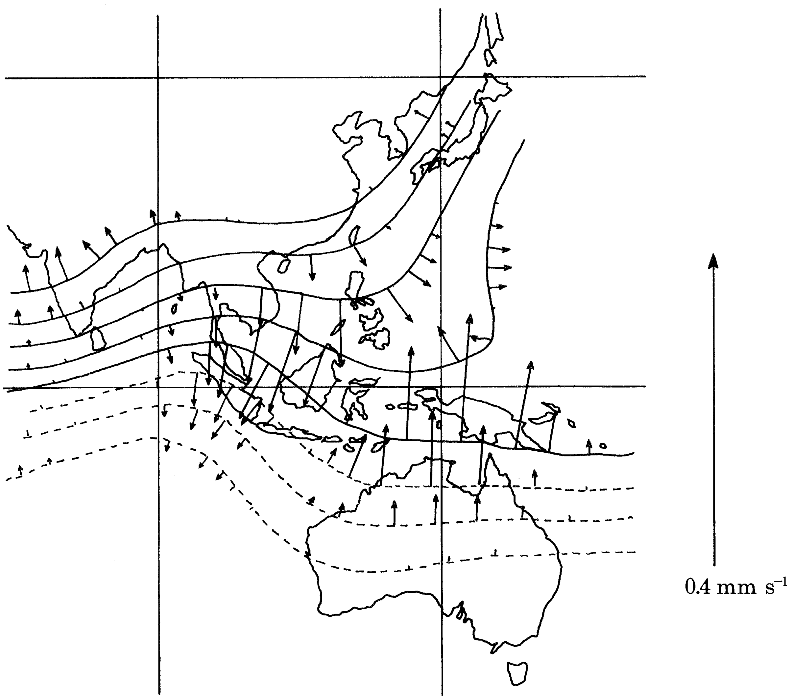

Figure 2. An enlargement from figure 1. The northward and southward flow is significant and demonstrates that the secular variation cannot be accounted for by purely azimuthal flow.

Clearly we are seeing something more complicated than simple westward drift. There is a significant north–south flow over Indonesia (enlarged, figure 2, cf. Whaler, this symposium) and southern Africa. The field maximum in North America shows eastward flow, and whatever the flow along the contour there could never be pure westward flow. To be toroidal the flow should satisfy

$$\oint_{\partial S_i} v_n \, dl = 0,$$

where ∂S_i is a contour of B. This condition implies that as much flow goes into a surface as goes out, and it can be reviewed by looking at the arrows on the maps. The condition is quite well satisfied over North America and the null-flux curve off South Africa. The Mongolian anomaly, reported by Yukutake (1968) to have grown in amplitude, without drifting, in historical times, also seems to exhibit northward flow with little upwelling. The South American null-flux curve suggests some upwelling, and the most significant region is the negative field anomaly over South Africa. This whole area suffers from low field gradients and is close to a region of very rapid secular change, which may have made the flow determination inaccurate.

4. Steady motions

The induction equation relates instantaneous core flow to B and $\dot{B}$, and no assumption of steady flow is implied. However, observations are of B rather than $\dot{B}$, and the time derivative is usually estimated by differencing measurements of B separated by several years. This procedure contains an implied assumption of steady flow. Integrating (2) with respect to time from t to $t+\Delta t$ gives

$$[B(t+\Delta t)-B(t)]\,\Delta t^{-1} = -\nabla\cdot(\boldsymbol{v}\bar{B}),$$

where $\bar{B} = \int_t^{t+\Delta t} B\,\mathrm{d}t\Delta t^{-1}$. If $\boldsymbol{v}$ is not steady then a shorter Δt needs to be employed. The assumption may not be valid for the 'jerk' in secular variation that occurred around 1970.

Steady flow on some timescale must be assumed if we are to make progress. If the assumption holds on a longer timescale then further information about $\boldsymbol{v}$ becomes available. One way of looking at this is to consider secular acceleration. Differentiating (2) gives

$$\ddot{B}+\nabla\cdot(\boldsymbol{v}\dot{B}) = 0,$$

and knowing $\ddot{B}$, $\dot{B}$ gives a completely new set of results for $\boldsymbol{v}$.

Secular acceleration is very difficult to estimate and a better approach is to carry out an analysis of secular variation at widely separated periods in time, say 1910–25 and 1960–75. Suppose we have two such analyses assuming toroidal flow. These will give components of flow perpendicular to the contours of B for both epochs, but in most places the contours will have changed during the interval of 50 years separating the two analyses. With two sufficiently *different* components of $\boldsymbol{v}$, we have found the entire flow. Furthermore there will be points where the contours have not changed and where the *same* component of $\boldsymbol{v}$ will be calculated at the two times. These points furnish a test for steady flow.

It remains to be shown that the data are sufficiently precise at earlier times for this test to be carried out. Success will also depend on whether the time interval of 50 years can provide a sufficient change in the field pattern.

5. Discussion

I have proposed an approach for finding core motions from magnetic observations. A simplifying hypothesis is proposed, tested against the observations, and then adopted. Ultimately this procedure will lead to a velocity model for the core flow. It is then essential to disover if this flow gives an adequate fit to the data. The next stage is to compute strictly toroidal flows and find how well they fit the model. Ultimately, however, the question of whether or not there is significant poloidal flow can only be answered by constructing toroidal motions that fit the actual measurements rather than the derived field model, because only the measurements have error bounds.

In fitting actual measurements we will be constraining the model to satisfy the conditions (4) and (5). These constraints may be very useful in improving models based on rather small datasets. The integrals over null-flux curves give five independent constraints on the secular variation coefficients, effectively reducing the number of unknowns by five.

B. A. Hobbs and L. Shure developed the field and secular variation models. This work is partially supported by N.E.R.C. grant no. GR3/3475.

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Discussion

R. Hide, F.R.S. (*Meteorological Office, Bracknell, U.K.*). Professor Runcorn has asked me to comment on these two important papers by Dr Whaler and Dr Gubbins. I have little to add to the remarks I made at Professor Runcorn's request during the discussion of Dr Kovacheva's paper. Dr Whaler and Dr Gubbins find fairly pronounced north–south components of core motions. This is what one might expect if transverse waves are present.

M. F. Osmaston (*The White Cottage, Sendmarsh, Ripley, Surrey, U.K.*). Because of the high electrical conductivity Dr Gubbins's inferred apparent dearth of upwelling motions must relate only to motions in the extreme outermost part of the core. What does Dr Gubbins regard as the effective depth of the data details upon which his inference depends? Is this really deep enough to tell us anything at all about the core's deep convection pattern, particularly if the near-surface flow is affected by core–mantle coupling torques?

D. Gubbins. Results derived by assuming high electrical conductivity apply only to core motions at the top of the free stream immediately below the boundary layer. The boundary layer may be some tens of kilometres thick. The assumption of high conductivity is only valid if the field does not change rapidly over length scales of 100 km or so, and the procedure is only self-consistent if the results are valid to this sort of depth. The results tell us nothing about convection. Also those motions with inductive effects that are comparable with diffusion, namely those associated with the dynamo generation of the field, must also be negligible if the procedure is to be self-consistent.